Quiz 2

MTH 403, Semester 1, 2021-22

Duration: 55 minutes

Maximum Points: 30

Directions: While writing solutions, please ensure that you sufficiently explain and justify all intermediate arguments leading to any conclusions that you may draw along the way. Each statement (or argument) in your solution must be clearly explained, and must be devoid of any logical fallacies or gaps.

1. Let $\{f_n\}$ be a sequence of continuous functions on $[0,1]$ such that $f_n \to f$ uniformly on $[0,1]$. [5+10]

   (a) Let $g_n(x) = \int_{[0,x]} f_n$ and $g(x) = \int_{[0,x]} f$. Are g_n and g continuously differentiable on $[0,1]$. Explain why or why not.

   (b) Show that $g_n \to g$ uniformly on $[0,1]$.

2. Determine whether the following statements are true or false. If true, give a proof of the statement, and if false, provide a counterexample to the statement. [5+5+5]

   (a) If $f(x,y) + g(x,y)$ is integrable over a rectangle R, then both $f(x,y)$ and $g(x,y)$ are integrable over R.

   (b) If $|f(x,y)|$ is integrable over a rectangle R, then so is $f(x,y)$.

   (c) If $f(x,y)$ is integrable over a rectangle R and $\iint_R f^2 = 0$, then $f(x,y) = 0$ on R.